Pearson Edexcel

Examiners' Report
Principal Examiner Feedback

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Pearson Edexcel International GCE
In Further Pure 3 (WFM03)
Paper : 01 Further Pure F3

General

This paper proved to be a good test of student knowledge and understanding. It discriminated well between the different ability levels. There were many accessible marks available to students who were confident with topics such as conic sections, hyperbolic functions, integration, vectors and matrices.

Report on Individual Questions

Question 1

Although many fully correct solutions were seen to this question on the application of vector products to areas and volumes, there was a mixed response overall. In particular, the last mark in part (b) was not widely scored.

In part (a), the expected method was fairly well known but the usual slips were seen in obtaining the relevant vectors and in calculating a cross product. Some attempted to use position vectors, which was only viable using the rarely seen alternative of a sum of three vector products. Many students were unsure about what to do after obtaining the normal vector. A small number of attempts used the cosine rule but there were often slips and many could not find an exact area having found the cosine of one of the angles. This also applied to the smaller number who used a scalar product to find $\cos\theta$. It was quite rare to see an attempt that found one of the altitudes of the triangle.

Part (b) was slightly more demanding and omitted by some. Most attempts did find a relevant vector, but errors in forming the scalar triple product followed. The required $\frac{1}{6}$ was occasionally missing or wrong (usually $\frac{1}{2}$ or $\frac{1}{3}$). Those who obtained the correct expression in terms of k often neglected to use the required modulus signs. Confusion continues to be seen between the modulus of an algebraic expression and the magnitude of a vector.

Question 2

This question on hyperbolic functions saw good scoring on the whole. It was unusual to see attempts that scored no marks since most could obtain an acceptable derivative in part (a), although a recurring slip was the omission of the “2”. Converting to an expression in sinh $2x$ and cosh $2x$ generally went well although there were some circuitous routes and some weak attempts. A particularly common error here was to write $2\text{sech}^2 2x$ as $\frac{1}{2\cosh^2 x}$ instead of $\frac{2}{\cosh^2 x}$. Using exponential definitions at any stage was not a sensible strategy here.

The method mark in part (b) was widely scored, although many chose not to use the logarithmic form of arsinh and used exponential form which required rather more work. Those who had the correct value of p from part (a) were usually fully correct here with only a very small number losing the last mark by offering an unrejected extra invalid solution from a quadratic in e^{4x}.

Question 3

There was reasonable scoring in this question on inverting a matrix. Those who were confident with the method tended to score all six marks. In part (a) it was unusual to see responses where the student did not know what “singular” meant and most obtained the correct determinant and solved the resulting quadratic correctly. Most determinants were processed conventionally although the rule of Sarrus was occasionally seen.

The usual mistakes were seen in part (b), with some students omitting at least one of the three steps needed to obtain the adjoint matrix. However, it was not common to see the determinant not applied to the adjoint or applied incorrectly. A few students who knew the method succumbed to slips with one or two of the elements.

Question 4

This question on integration by substitution with a hyperbolic function proved to be quite discriminating and not many fully correct solutions were seen. The first three marks were scored widely – most students were able to use the substitution but some were unable to simplify the $(x^2 - 16)^{\frac{3}{2}}$ with $x = 4\cosh\theta$. Correctly integrating $\frac{1}{16\sinh^2\theta}$ proved a challenge and even those who remembered this result often fell foul of slips when substituting back to get an expression in x.

Question 5

Although this type of question on diagonalising a matrix is common to this paper, it was fairly unusual to see fully correct solutions although plenty of marks were accessed. In part (a) some students appeared to think they were being asked to verify that 8 was an eigenvalue and so launched into solving the characteristic equation. Most recovered, relabelling their work, but some ended up confusing the methods of obtaining eigenvalues and eigenvectors. The correct eigenvector was widely seen although there were errors in forming and solving the systems of simultaneous equations. Attempts by finding the cross product of two rows of $\mathbf{M} - \lambda\mathbf{I}$ were very rare.

In part (b) there were few incorrect attempts at the determinant of $\mathbf{M} - \lambda\mathbf{I}$ but many were unable to produce the correct cubic. A few solutions confidently maintained $(6 - \lambda)$ as a factor which simplified the algebra. Those who obtained the correct cubic invariably proceeded to find the two other eigenvalues.

Some did not attempt part (c) but those who did tended to pick up some marks. Slips in finding the other two eigenvectors were fairly rare but a common error was not to normalise them before forming **P**. It was very uncommon to see a "zero eigenvector" involved but it remains the case that some students are not aware that **D** is the diagonalised matrix of eigenvalues. Those who attempted to find **D** by actually performing the multiplication of $\mathbf{P}^{\mathrm{T}}\mathbf{MP}$ were rarely successful.

Question 6

Reduction formulae questions which don't involve powers of trigonometric or hyperbolic functions often prove demanding but there was plenty of creditable work here. Those unable to make progress with the proof in part (a) tended to pick up at least some of the marks in part (b). The key to most reduction formulae proofs is to spot the required "split". Although an alternative route was possible, the overwhelming majority of successful attempts applied $x^n =$

$x^{n-1} \times x$ to the integral. The pitfall of attempting to use $u = x^n$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = (x^2+3)^{-\frac{1}{2}}$ in the parts formula was quite common. Those who did identify the correct initial split generally scored the first three marks but the next step of applying $(x^2+3)^{\frac{1}{2}} = (x^2+3)(x^2+3)^{-\frac{1}{2}}$ was elusive for many. Those able to do this generally proceeded to give a fully correct proof. Some unconvincing attempts were seen which tried to work backwards from the given answer.

In part (b), most who made an attempt came away with some marks and a fully correct expression in x was quite common – usually via a "top-down" rather than a "bottom-up" approach. Weaker attempts were unable to determine I_1. Successful efforts which included trying to find I_3 directly were very rare indeed. A few errors were seen when attempting to obtain the answer in the required form. A small number neglected to include the given answer's "+ k".

Question 7

This question on points and planes in vector space was a fairly challenging one and although some students were clearly ill-prepared for it, there were an encouraging number of correct solutions here.

Part (a) relied upon taking the cross product of two vectors in the plane and many were able to find the required normal. There was some confusion seen with points and planes, as well as sign slips extracting the point and direction from the equation of the given line. Since this was a "show that" question it was important that the origin of the "5" (or "35") was made clear and this wasn't always the case. There were viable alternative approaches to obtaining the plane equation but they were not common.

Part (b) saw a lot of success, particularly for those who used the result from the formula book. Those who used $\left|\frac{\overrightarrow{PQ}.\,\mathbf{n}}{|\mathbf{n}|}\right|$ were also generally successful. The parallel plane method, although seemingly widely-taught, had more mixed results, with some students unable to combine the $\frac{5}{7}$ and the $\frac{-9-2k}{7}$ correctly to reach the given answer.

The marks in part (c) were not awarded often, although those who were well practised in finding the distance of a point from a plane could usually form a correct equation. Handling the modulus signs proved an issue for some, with many only being able to find one solution. Those who squared both sides were more likely to get two answers although poor squaring occasionally led to the wrong quadratic. A few responses offered more than 2 values for k, the result of using a misguided "regions" method to deal with the modulus or from solving four linear equations rather than two.

Question 8

This question on the length of an arc saw a mixed response. There was certainly plenty of scoring in part (a) but the integral in part (b) proved to be demanding.

In part (a), the correct derivative was widely seen although the minus sign was occasionally lost. Most then used the formula correctly but some showed only cursory working which often led to lost marks. A small number did not reach the integral as printed on the question paper – omitting the limits or the "dx".

The best route in part (b) was to first deal with the improper fraction. Those who spotted this usually went on to recognise the integrand and score all the marks. However, there were various different strategies which attempted to write the fraction in a different form but this never led to an expression that could be completely integrated with ease. This often led to students giving up or incorrect attempts using substitutions, parts or invalid partial fractions. Although the original integral could be dealt with by hyperbolic and trigonometric substitutions, these were generally demanding routes and correct attempts using these methods were rarely seen.

Question 9

The last question on an ellipse was a good source of marks for many although there were some rushed attempts from students who had not managed their time well.

In part (a), the equation of the normal to an ellipse was required and a large number of correct proofs were seen. The easiest method of parametric differentiation was not always chosen and many implicit or even explicit differentiations were seen. A few of those differentiating implicitly forgot to exchange the "1" for "0". Most attempts obtained $\frac{dy}{dx}$ in terms of θ and the correct perpendicular gradient rule and a correct straight line method invariably followed. The inelegant $y = mx + c$ approach was seen a number of times but usually did not lead to any lost marks.

The two marks in part (b) were quite accessible. Correct formulae for eccentricity and the x-coordinate of the focus were widely used although errors in substitution were seen (such as replacing the a in ae with 25 instead of 5). The question clearly stated that point F was on the positive x-axis so it was careless of students to give an answer of $F(\pm 3, 0)$.

Part (c) was more demanding and the last mark in particular was infrequently awarded. There were very few errors in obtaining the correct x-coordinate of Q and most attempts proceeded to apply Pythagoras to find PF. Most continued to get the correct quadratic in PF^2. The last mark required the $\frac{3}{5}$ or $\frac{9}{25}$ to be clearly isolated within either a correct expression for $\frac{|QF|}{|PF|}$ or $\frac{QF^2}{PF^2}$. A few using the first of these approaches had a ratio where the numerator and denominator were of different signs. However, a substantial amount of well-constructed and well-presented fully correct proofs were seen.